AIAA STUDENT PAPER
MARCH 1977

A DESIGN APPROACH TO PROPELLER-TYPE WINDMILL THEORY

By
Walter A. Lounsbery
Undergraduate Aerospace Engineering Student
Department of Mechanical & Aerospace Engineering
University of Missouri - Rolla
Rolla, Missouri 65401

1977 AIAA REGION V STUDENT CONFERENCE

St. Louis, Missouri
March 30 - April 1, 1977

A DESIGN APPROACH TO PROPELLER-TYPE WINDMILL THEORY

Walter A. Lounsbery*
University of Missouri-Rolla
Rolla, Missouri 65401

Abstract

A practical, design oriented theory for propeller-type windmills is developed. By substituting pitch as a variable in the familiar vortex theory, the equations for power, torque, and thrust are simplified. Optimization of design (maximizing power output) by variation of blade angle is explored. This leads to the postulation of a general solution for power from a windmill with blades of optimum twist, given that a particular airfoil is used. This General Windmill Equation is solved for the GA(W)-1 airfoil. Indications that high tip speed ratios are desirable for high efficiencies also show the need for Mach effect corrections. A correction for the GA(W)-1 is developed and the possibility of Mach braking or use of supercritical airfoils is discussed. The theory developed leads to design tools embodied in two main computer programs: an optimized blade angle design program; and a windmill evaluation program with correction for Mach effect.

---

*This paper presented at AIAA Region V Student Conference in St. Louis, Missouri, March 31, 1977. (Undergraduate Division)

Nomenclature

a_o - lift slope, per radian

b - number of blades

c - blade chord, ft

C - downwash coefficient

C_D - coefficient of drag of the airfoil

C_L - coefficient of lift of the airfoil

C_{Lo} - C_L not corrected for induced effects

C_P - elemental power coefficient

C_Q - elemental coefficient of torque

C_T - elemental coefficient of thrust

dD - elemental drag, lb

F_P - power factor, ft^3

F_Q - torque factor, ft^3

F_T - thrust factor, ft^2

dL - elemental lift, lb

$\frac{L}{D}$ - lift-to-drag ratio of the airfoil

n - rotation speed, rev/sec

p - pitch, ft

P - power output, ft-lb/sec

Q - torque, ft-lb

r - radius, ft

r_o - blade inner radius, ft

r_1 - blade tip radius, ft

T - thrust, lb

V_i - induced downwash velocity, ft/sec

V_R - induced air velocity, ft/sec

V_{R_o} - free stream air velocity, ft/sec

V_w - wind velocity, ft/sec

α - angle of attack, radians

α_o - free stream angle of attack, radians

β - geometric pitch angle, radians

ϕ - induced velocity effective pitch angle, radians

ϕ_o - free stream effective pitch angle, radians

ρ - air density, slug/ft^3

θ - induced angle, radians

Introduction

Wind energy conversion systems (WECS) were in use even before Hero of Alexandria described a windmill powered pipe organ[1]. WECS have usually been applied to more practical problems, however. Until World War I, tens of thousands of windmills were in use throughout Europe for grinding grain, draining land, or electrical power generation. Up to 18 kW power generation was realized even at the turn of the century[2]. Many decades of practical experience have been accumulated on various forms of WECS.

As long as the windmill has been around, and as extensively as it has been used, there seems to be a gap between basic theory and its application, especially in the literature. One of the latest substantive publications devoted to windmill theory[3] does little more than review much older literature, without really indicating how the basic theory translates into practice. This gap between theory developed and practical experience, it seems, needs to be filled before any of the many windmill theories can be proved by past experience.

This paper is an effort to bridge the gap between theory and experience for propeller-type windmills. The basic vortex theory for windmills is developed into practical form, and embodied in two main computer programs.

Propeller-Type Windmill Theory

Basic windmill theory can be derived in a similar manner to the vortex theory of propellers[4]. That is, all sections of the blade are assumed independent of all other sections. Since an airfoil section producing lift imposes some downwash velocity, V_i, on the airstream, the actual situation at any radius becomes that shown in Figures 1 and 2. This induced velocity subtracts an angle θ from the free stream angle of attack α_o, changing the actual angle of attack. Standard vortex theory gives the induced angle, θ, as

$$\theta = \frac{\alpha_o}{1 + \frac{8\pi r \sin\phi}{a_o bc}} \quad \text{where } \alpha_o = \alpha + \theta \tag{1}$$

Utilizing this and the vector relations summarized in Figure 1, it is possible to find the forces on the blade section. Differential thrust, torque, and power are then

$$dT = (C_L \sin\phi + C_D \cos\phi)(\cos\theta)\frac{b}{2}\,\rho c V_w^2 \sec\phi_o dr \tag{2}$$

$$dQ = (C_L \cos\phi - C_D \sin\phi)(\cos\theta)\frac{b}{2}\,\rho r V_w^2 \sec\phi_o dr \tag{3}$$

$$dP = 2\pi\eta dQ \tag{4}$$

While this theory leads to reasonable accurate predictions of performance, in form it is somewhat unsatisfactory. Many of the variables appear unrelated, and solutions require many conditions or restraints.

The differences in varuables are most notable in the angles (ϕ, ϕ_o, θ) and in the relation between torque and power. If these could be related to each other, some new facts might come to light.

Relating these variables is possible by introducing a new variable, pitch. By defining pitch as the linear amount of wind past the blades in each revolution of the blades, a vector relation results which has wide application. In symbols

$$p = \frac{V_w}{\eta} \tag{5}$$

Pitch, even though it carries units of length, is an important operating characteristic, just as radius or chord are important operating characteristics.

Solving for the induced angle

$$\theta = \frac{\alpha_o}{1 + \frac{16\pi^2 r^2}{a_o bc((2\pi r)^2 + p^2)^{1/2}}} , \tag{6}$$

This may be modified by defining a downwash coefficient, C, involving the operating characteristics of the blade

$$C = \frac{16\pi^2 r^2}{bc[(2\pi r)^2 + p^2]^{1/2}} \tag{7}$$

Giving

$$\theta = \frac{\alpha_o}{1 + \frac{C}{a_o}} \tag{8}$$

This form is accurate only on the linear portion of an airfoil's lift curve (assuming one exists). Remembering the definition of lift curve slope, and applying it to (6), we have

$$\theta = \frac{C_L}{C} \tag{9}$$

or in reference to the coefficient of lift before downwash correction,

$$\theta = \frac{C_{Lo}}{(\frac{C_{Lo}}{\alpha_o} + C)} \tag{10}$$

These two forms are much more useful than the standard expression.

The formuli for thrust, torque, and power are then

$$dT = (C_L \sin\phi + C_D \cos\phi)(\cos\theta)\frac{b}{2}\rho(\frac{V_w}{p})^2[(2\pi r)^2 + p^2]^{1/2} dr \tag{11}$$

$$dQ = (C_L \cos\phi - C_D \sin\phi)(\cos\theta)\frac{b}{2}\rho(\frac{V_w}{p})^2 r[(2\pi r)^2 + p^2]^{1/2} dr \tag{12}$$

$$dP = 2\pi(\frac{V_w}{p}) dQ \tag{13}$$

where

$$\phi = \tan^{-1}\left(\frac{2\pi r}{p}\right) + \theta \tag{14}$$

Elementary coefficients of thrust, torque, and power may be defined from these as

$$C_T = (C_L \sin\phi + C_D \cos\phi)\cos\theta \tag{15}$$

$$C_Q = C_p = (C_L \cos\phi - C_D \sin\phi)\cos\theta \tag{16}$$

Setting the power coefficient to zero, and assuming small θ

$$r_{max} = \frac{p(\frac{L}{D})_{max}}{2\pi} \tag{17}$$

or

$$r_{max} = \frac{V_w(\frac{L}{D})_{max}}{2\pi R_{tip}} \tag{18}$$

If the operating characteristics of the blade are separated from wind velocity and air density, an interesting and useful basic relation results. For the entire blade

$$T = \rho\,\frac{V_w^2}{p^2}\int_{r_o}^{r_1}(C_L \sin\phi + C_D \cos\phi)\cos\theta\frac{b}{2}[(2\pi r)^2 + p^2]^{1/2}dr \tag{19}$$

or

$$T = \rho V_w^2 F_T \tag{20}$$

Where F_T is a function of pitch for a given windmill. Similarly

$$Q = \rho V_w^2 F_Q \tag{21}$$

$$P = \rho V_w^3 F_P \tag{22}$$

These relations indicate important parameters, and a simple means of plotting their variation.

Design Theory

The basic vortex theory has already been extensively modified to make it more practical. However, some specific performance parameters should be discussed, as well as optimization of propeller-type windmills.

The main performance parameters are the thrust, torque, and power factors, which give thrust, torque and power at any wind speed and rotative speed. Bending moment at the hub may also be found. Efficiency is generally expressed in terms of "Betz" efficiency, or the fraction of available power captured by the windmill. The maximum available power, according to Betz[5], is

$$P_{max} = \frac{16}{27}\pi (R_{max})^2 (\frac{1}{2}\rho V^3) \qquad (23)$$

However, many later studies, some using more accurate flow modeling, arrived at quite different results[6]. Some have indicated the maximum power available as high as 70% of the power in the wind[7]. Betz's results also seem doubtful since some windmills have approached 90% Betz efficiency without the aid of today's efficient airfoils.

Thus it seems best to simply reference efficiency to the power in the wind.

The performance parameter of immediate interest, of course, is power. There are only four major variables effecting power, as can be seen from (12) and (13): the airfoil, radius, chord, and pitch. Varying airfoil characteristics, that is selecting an airfoil (the GA(W)-1 in the examples[8]) and varying its angle of attack, gives plots similar to Figure 3. This is typical of all airfoils and conditions.

It can be seen that there is only one angle of attack for a given airfoil which produces the most power. While this is intuitively obvious, a proof will be given later. Only this angle of attack need be considered, and it may be found by plotting power output versus angle of attack. This leaves three variables (pitch, radius and chord), and typical variations of power with these variables can be seen in Figures 4, 5, and 6.

This leads to an interesting postulation, based on evidence of a few graphs. Since each variable produces a well-defined function of power, then there ought to be a general equation of power for a windmill of given airfoil operating at maximum power output. In other words, there is a function

$$P = F(r,c,p) \qquad (24)$$

If this function were continuous and differentiable and could be found, then by suitably restraining the variables, the optimum windmill could be produced directly from theory. There would be only one "best" design and it could be found without any trial and error methods.

This method is, of course, subject to restrictions which become important under some conditions. The basic assumption of independent sections along the blade breaks down towards the tip of the blade. Looking at Figure 3, high angles of attack and blade loadings indicate the formation of strong tip vortices. While this may be neglected or within the bounds of error, the shortcoming should be realized. It is interesting to note that vortex suppressors for wings could be applied to windmills, in the form of tip plates, Hoerner tips[9], or winglets[10].

The blade tips encounter another difficulty as well. From Figure 4 it can be surmised that low pitches are desirable for high power outputs. Unfortunately, this leads to high tip speeds, approaching the transonic even in moderate winds. Correction for Mach effects on the blade is clearly necessary for some designs. For example, the NASA-ERDA demonstration windmill at Sandusky, Ohio[11], reaches a maximum Mach number at the tip of 0.236. Presumably it was designed that way to avoid Mach effects, since the NASA-ERDA windmill operates at a much higher pitch than optimum.

Even though Mach effects can create quite a problem, both aerodynamically and aeroelastically, some use could be made of the phenomenon. One of the worst problems with large windmills is overspeed in high winds leading to overloading of the structure. By designing a windmill to operate close to the transonic region, drag divergence on the airfoil could prevent overspeeding (Mach braking). Designing out the accompanying undesirable aeroelastic effects would certainly be a challenge. Alternatively, supercritical airfoils could be used to alleviate Mach effects[12] and increase power output. In Appendix A an approximate correction for Mach effects on the GA(W)-1 is shown.

Obtaining maximum power through the theory already developed still has merit once its restrictions are recognized. To begin with, optimum blade angle at any radius can be found through the elementary power coefficient

$$C_P = (C_L \cos\phi - C_D \sin\phi)\cos\theta \qquad (25)$$

Recognizing that there is no unique solution for maximum C_P, airfoil characteristics must be substituted into the equation to find the maximum C_P for any given airfoil. A relation between C_L and C_D of the airfoil is necessary, and may be approximated by a third order polynomial over a restricted range for the GA(W)-1

$$C_L = 229.858D_C^{\ 3} - 178.4C_D^{\ 2} + 27.5256C_D + 0.412355 \qquad (26)$$

$$0.01 < C_D < 0.1$$

Substituting into (25) and setting the derivative equal to zero gives

$$0 = aC_D^{\ 4} + bC_D^{\ 3} + cC_D^{\ 2} + dC_D + e \qquad (27)$$

$$a = 205033\sin\phi$$

$$b = -(88961.04\sin\phi + 689.574\cos\phi)$$

$$c = [(689.574)(C)\cos\phi + 14447.35\sin\phi + 256.8\cos\phi]$$

$$d = -[(356.8(C) + 689.574)\cos\phi + 610.53\sin\phi]$$

$$e = C[27.5256\cos\phi - \sin\phi] - 11.3503$$

Assuming at this point that θ is very small, or negligible, the resulting biquadratic equation may be solved for the drag coefficient resulting in maximum C_P. The entire equation, though lengthy, defines optimum blade angle power output solely in terms of r, c, and p. The final solution is not shown here, since it is elementary from this point and takes more room than can be afforded.

Once the solution to the General Windmill Equation is obtained (for any given airfoil) the variables r, c, and p may then be subjected to restraints derived from economic factors, size limitations, structural and dynamic factors, and the desired power output. Blade area distribution is then solved, and some backtracking gives the β values along the blade.

Presumably this type of analysis is not limited to just the theory derived. It is possible to arrive at corrections for Mach effect, tip loss, Reynold's Number effects, and so on. However, each correction must necessarily be an approximate one, and cumulative error would wipe out any gains in accuracy after many corrections.

Design Tools

Several computer programs were developed to aid design and investigate windmill theory. These were written in VS BASIC as implemented on the University of Missouri IBM 370-168. Using the Time Sharing Option (TSO) enabled these programs to be designer-interactive, and allowed greater versatility than batch processing. These programs are found in Appendix B.

The first program (POLY) is a short polynomial solving and testing program which can be used to fit lift and drag curves for airfoils.

It was used to find ninth order polynomials approximating C_L versus α, C_D versus α, and a third order polynomial approximating C_L versus C_D for the GA(W)-1.

Programs TEST1 and TEST2 are simple programs that find pitch for a given RPM and wind speed, and find tip speed under given conditions, respectively.

WECS1 is an involved program which centers around a subprogram that optimizes blade angle for the GA(W)-1. The first half allows the user to vary any one parameter to find how power output changes. The second half finds the optimum blade angles given a linear or non-linear area distribution and a certain operating pitch. In effect it "designs" the blade twist. The user may then evaluate the design under a variety of operating pitches.

WEP is a windmill evaluation program which takes any design using the GA(W)-1 and evaluates performance parameters under a variety of conditions. A special feature is an approximate correction for Mach effect on the blade.

Conclusions and Recommendations

Several interesting conclusions can be drawn from the theory developed.

1. Airfoils with high lift to drag ratios at high coefficients of lift are necessary for maximum power output.
2. Low pitches are desirable for maximum power output.
3. Tip loss may be appreciable in some designs, leading to errors in theoretical predictions.
4. Mach effects are a problem with some designs, producing large errors in theoretical predictions.
5. Given a particular airfoil used, there is only one blade angle that gives maximum power output under specified conditions (radius, pitch, and chord).
6. There is a General Windmill Equation for propeller-type windmills describing power from a windmill with such an optimized blade.

The theory developed is in a practical form, but needs to be applied to existing test data to check its validity. Tip loss correction should be included in the WEP program for more accurate results.

References

1. Whittingham, Charles; The Pneumatics of Hero of Alexandria, London, 1851

2. Golding, E.W.; The Generation of Electricity by Wind Power, Philosophical Library, Inc., New York, 1956, pp. 15,16.

3. Wilson, Robert E., and Lissaman, Peter B.S.; Applied Aerodynamics of Wind Power Machines, Oregon State University, May 1974.

4. Dommasch, Daniel O., et. al; Airplane Aerodynamics, 4th Ed., Pitman Publishing Corp., New York, 1967, pp. 222-228.

5. Betz, A.; "Windmills in the Light of Modern Research", NACA TM-474, p. 27, August 1928.

6. Sabinin, G. Kn.; "The Theory of an Ideal Windmill", Transactions of the Central Aero-Hydrodynamical Institute, Moscow, 1928.

7. "Final Report on the Wind Turbine", Research Report PB25370, Office of Production, Research, and Development, War Production Board, Washington, D.C., 1946.

8. McGhee, Robert J., and Beasley, William J.; "Low Speed Aerodynamic Characteristics of a 17-Percent Thick Airfoil Section Designed for General Aviation Applications", NASA TN D-7428, December 1973.

9. Hoerner, Sighard F.; Fluid Dynamic Drag, Published by the Author, 1965, pp. 7-4 to 7-11.

10. Pierpont, P. Kenneth; "Bringing Wings of Change", Astronautics and Aeronautics, Vol. 14, No. 2, Feb. 1976, pp. 19-31.

11. "Wind Energy Developments in the 20th Century", Lewis Research Center, NASA, Cleveland, Ohio, 1975, Revised, p. 9.

12. Whitcomb, Richard T., and Clark, Larry R.; "An Airfoil Shape for Efficient Flight at Supercritical Mach Numbers", NASA TM-1109, May 1965.

13. Abbott, Ira H., and Von Doenhoff, Albert E.; Theory of Wing Sections, Dover Edition, Dover Publications, Inc., New York, 1959, pp. 276-287.

Appendix A

Since data was not available on Mach effects on the GA(W)-1 performance, a correction was developed based on an airfoil of similar thickness ratio, the NACA 2315[13]. It is impossible to say how accurately this data applies to a thick laminar flow section like the GA(W)-1, but it was felt that some approximate correction could give an idea of just how Mach effects would change the power output of a particular design, even though the actual prediction would be quantitatively in error.

Coefficient of lift correction is based on C_L before correction. Figure 7 shows C_L versus Mach number for the NACA 2315. Simplifying the plot somewhat, variation of C_L can be assumed to occur as in Figure 8. C_L is unchanged up to M_o, where it describes a sinusoid superimposed on a negative slope line. M_o may be approximated by

$$M_o = (-0.25)C_L + 0.525 \tag{A-1}$$

The line is a simple matter to fit, having the form

$$y = [(-0.339105)C_L - 0.34387]\ (M-M_o) + C_L \tag{A-2}$$

The first half period of the sinusoid occurs between M_o and two thirds of the Mach value between M_o and M equals 0.95, the range of the correction function. Amplitude is constant such that

$$(C_L)_{corr} = y + (0.15)\sin\left[\left(\frac{3\pi}{2(0.95-M_o)}\right)(M-M_o)\right]$$

$$M_o < M < \frac{2}{3}\left(\frac{M - M_o}{0.95-M_o}\right) \tag{A-3}$$

Over the last third of the correction range, the amplitude of the sine function varies with C_L. A linear fit gives

$$(C_L)_{corr} = y - (0.5C_L + 0.05)\sin\left[\left(\frac{3\pi}{(0.95 - M_o}\right)\left(\frac{M + 1.9 - M_o}{3}\right)\right]$$

$$\frac{2}{3} \le \frac{M - M_o}{0.95 - M_o} \le 1 \tag{A-4}$$

Drag divergence is essentially a cubic function starting at a different M_o (Figure 9)

$$M_o = (-0.0125)(\alpha)_{degrees} + 0.6 \qquad \text{(A-5)}$$

Variation of the factor in the cubic may be approximated in a linear fashion, giving

$$(C_D)_{corr} = [(-9.62 \times 10^{-5})(\alpha)_{degrees} + (3.149 \times 10^{-3})] [10(M-M_o)]^3 \pm C_D \qquad \text{(A-6)}$$

APPENDIX B

```
**************POLY**************
10 PRINT 'ORDER OF POLYNOMIAL?'
20 INPUT N
30 Q1=N+1
40 DIM X(Q1,Q1),C(Q1,1),B(Q1,1),Y(Q1,Q1)
50 MAT X=CON
60 FOR I=1 TO Q1
70 PRINT ' IND. VARIABLE?'
80 INPUT X(I,N)
90 Z=X(I,N)
100 FOR J=N-1 TO 1 STEP -1
110 X(I,J)=X(I,J+1)*Z
120 NEXT J
130 PRINT 'DEP. VARIABLE?'
140 INPUT B(I,1)
150 NEXT I
160 MAT Y=INV(X)
170 MAT C=Y*B
180 PRINT 'COEFFICIENTS'
190 FOR I=1 TO Q1
200 PRINT C(I,1),'POWER OF VARIABLE IS ';Q1-1
210 NEXT I
220 PRINT
230 PRINT
240 PRINT 'TEST FIT OF POLYNOMIAL'
250 PRINT
260 PRINT 'INDEPENDENT VARIABLE?'
270 INPUT X1
280 FOR I=1 TO Q1
290 X2=C(I,1)*X1**(I-1)
300 NEXT I
310 PRINT
320 PRINT 'INDEPENDENT VARIABLE IS ';X2
330 GO TO 250
340 END
```

```
***************TEST1***************
10 PRINT 'ENTER WIND VELOCITY, MPH'
20 INPUT V
30 PRINT 'ENTER DESIRED RPM'
40 INPUT R
50 V =V*88/60
60 R=R/60
70 P=V/R
80 PRINT 'DESIRED PITCH IS ';P
85 GO TO 10
90 END
END OF DATA
end
NOTHING SAVED
ENTER SAVE OR END-
end
READY
edit test2 vsbasic old
DATA SET TEST2.VSBASIC NOT LINE NUMBERED, USING NONUM
EDIT
top
insert
INPUT
***************test2***************

EDIT
1
***************TEST2***************
10 PRINT 'ENTER RPM'
20 INPUT R1
30 PRINT 'ENTER RADIUS'
40 INPUT R
50 PRINT 'ENTER WIND VELOCITY'
60 INPUT V
65 P=V*88/R1
66 PRINT 'PITCH IS ';P;' FEET'
70 V=V*88/60
80 V1=(V/P)*2*ATN(1)*4*R
90 V2=SQR(V1**2+V2**2)
100 PRINT 'TIP SPEED IN FT/SEC IS ';V1;' FT/SEC'
110 PRINT 'OR IN MPH ';V1*60/88;' MPH'
120 GO TO 10
130 END
```

```
************WECS1**************
3 DIM U(3,40)
5 READ C0,C9,C8,C7,C6,C5,C4,C3,C2,C1
6 READ B0,B9,B8,B7,B6,B5,B4,B3,B2,B1
10 PRINT 'THIS PROGRAM FINDS THE OPTIMUM BLADE ANGLE'
20 PRINT 'FOR A WINDMILL USING THE GA(W)-1 AIRFOIL'
30 PRINT 'AND A GIVEN PITCH, CHORD, AND RADIUS.'
40 PRINT 'IT GIVES THE POWER AND FORCE FACTORS FOR THIS ANGLE.'
50 PRINT '  YOU HAVE THE OPTION OF VARYING ONE PARAMETER IN ANY RANGE'
55 PRINT 'OR, BY SPECIFYING ''DESIGN'', DESIGN AND'
56 PRINT 'EVALUATE A COMPLETE WINDMILL'
60 PRINT
70 PRINT 'WHAT VARIABLE DO YOU WISH TO VARY?'
80 PRINT 'TYPE PITCH,CHORD, RADIUS, OR NO FOR NO VARIATION'
90 INPUT A$
100 IF A$='PITCH' THEN 250
110 IF A$='CHORD' THEN 370
120 IF A$='RADIUS' THEN 490
125 IF A$='DESIGN' THEN 950
130 PRINT 'YOU MAY SPECIFY THE OPTION LATER BY TYPING'
135 PRINT '-1 WHEN QUERIED FOR A PITCH VALUE'
140 PRINT 'TO TERMINATE, ENTER ZERO PITCH'
145 PRINT
150 PRINT 'PITCH?'
160 INPUT P
170 IF P=0 THEN 1930
180 IF P<0 THEN 60
190 PRINT 'RADIUS? CHORD?'
200 INPUT R,B
210 Z=1
220 GOSUB 600
230 GO TO 150
240 REM PITCH VARIABLE
250 PRINT 'PITCH FROM, TO, INCREMENT'
260 INPUT P3,P4,P5
270 IF P3=0 THEN 1930
280 IF P3 < 0 THEN 60
290 PRINT 'RADIUS AND CHORD'
300 INPUT R,B
310 Z=1
320 FOR P=P3 TO P4 STEP P5
330 GOSUB 600
340 NEXT P
350 GO TO 60
360 REM CHORD VARIABLE
370 PRINT 'CHORD FROM,TO,INCREMENT'
380 INPUT Q1,Q2,Q3
390 PRINT 'PITCH AND RADIUS'
400 INPUT P,R
410 IF P=0 THEN 1930
420 IF P<0 THEN 60
430 Z=1
440 FOR B=Q1 TO Q2 STEP Q3
450 GOSUB 600
460 NEXT B
```

```
470 GO TO 60
480 REM RADIUS VARIABLE
490 PRINT 'RADIUS FROM, TO, INCREMENT'
500 INPUT R1,R2,R3
510 PRINT 'PITCH AND CHORD'
520 INPUT P,B
530 IF P=0 THEN 1930
540 IF P<0 THEN 60
550 Z=1
560 FOR R=R1 TO R2 STEP R3
570 GOSUB 600
580 NEXT R
590 GO TO 60
600 IF Z<>1 THEN 640
610 PRINT USING 620,'PITCH','CHORD ','RADIUS','POWER','FORCE','ANGLE'
620 FORM SKIP2,X2,C,POS13,C,POS22,C,POS36,C,POS49,C,POS58,C
630 Z=0
640 P1=-10000
645 P2=-20000
650 A=R*ATN(1)*4
665 IF B>0 THEN 670
666 C=99999999
667 GO TO 680
670 C=4.0*((2*A)**2)/(B*SQR((2*A)**2+P**2))
680 FOR J=0 TO 30 STEP 0.5
690 F1=C0*J**9+C9*J**8+C8*J**7+C7*J**6+C6*J**5+C5*J**4
691 F1=F1+C4*J**3+C3*J**2+C2*J+C1
700 F2=B0*J**9+B9*J**8+B8*J**7+B7*J**6+B6*J**5+B5*J**4
701 F2=F2+B4*J**3+B3*J**2+B2*J+B1
710 U=2*A/P
720 T=F1/C
730 H=T+ATN(U)
740 P1=(F1*COS(H)-F2*SIN(H))*COS(T)
746 IF J < 14 THEN 760
750 IF P1<= (P2-0.08*ABS(P2)) THEN 810
760 IF P1<P2 THEN 800
770 P2=P1
780 J1=J
790 H1=H
795 T1=T
800 NEXT J
810 J=J1
820 F1=C0*J**9+C9*J**8+C8*J**7+C7*J**6+C6*J**5+C5*J**4
821 F1=F1+C4*J**3+C3*J**2+C2*J+C1
830 F2=B0*J**9+B9*J**8+B8*J**7+B7*J**6+B6*J**5+B5*J**4
831 F2=F2+B4*J**3+B3*J**2+B2*J+B1
840 P1=P2*((2*A)**2+P**2)*A*B/(2*P**3)
850 F3=(F1*SIN(H1)+F2*COS(H1))*COS(T1)*((2*A)**2+P**2)*B/(2*P**2)
860 PRINT
867 H1=H1*180/3.14159
870 PRINT USING 875, P,B,R,P1,F3,H1,J
875 FORM PIC(ZZZ.###),X3,PIC(ZZZ.###),X4,PIC(ZZ.###),X2,PIC(ZZZZZZZZ.##),
         X2,PIC(ZZZZZZZZ.##),X4,PIC(ZZ.###),X3,PIC(ZZ.#)
880 RETURN
890 DATA 3.524514E-12,-6.30564E-10,4.417507E-8,-1.499429E-6
900 DATA 2.492219E-5,-1.614988E-4,-4.649162E-4,8.909225E-3
```

```
910 DATA 7.790631E-2,5.672455E-2
920 DATA -8.995701E-12,1.026354E-9,-4.779145E-08,1.167798E-6
930 DATA -1.607709E-5,1.247505E-4,-5.00344E-4,8.673142E-4
940 DATA -4.346052E-4,1.005569E-2
950 PRINT
955 PRINT '***NOTE: ONLY THE AREA DISTRIBUTION, PITCH,'
960 PRINT 'AND RADIUS STATIONS CAN BE SPECIFIED.  THIS'
970 PRINT 'PROGRAM WILL STILL OPTIMIZE BLADE ANGLE FOR'
980 PRINT 'THE CONDITIONS SPECIFIED.***'
982 S1=0
983 S2=0
990 PRINT
1000 PRINT 'FOR A LINEAR AREA DISTRIBUTION ENTER ''LINEAR'''
1010 PRINT 'ENTER ''NON'' FOR A NON-LINEAR AREA DISTRIBUTION'
1020 INPUT B$
1030 IF B$ ='LINEAR' THEN 1365
1040 PRINT
1050 PRINT 'AFTER ALL THE DATA IS ENTERED ( RADIUS,CHORD),'
1060 PRINT 'ENTER 0,0 ON THE QUERY TO TERMINATE INPUT'
1070 PRINT
1080 PRINT 'PITCH?'
1090 INPUT P
1110 I=1
1120 PRINT 'RADIUS, CHORD?'
1130 INPUT U(1,I),U(2,I)
1140 IF U(1,I)=0 THEN1170
1150 I=I+1
1160 GO TO 1120
1170 R=U(1,1)
1172 I=I-1
1180 B=U(2,1)
1190 Z=1
1200GOSUB 600
1201 U(3,1)=(H1+J)*3.14159/180
1205 S1=P1
1210 S2=F3
1230 FOR W=2 TO I-1
1234 R=U(1,W)
1235 B=U(2,W)
1240 GOSUB 600
1241 U(3,W)=(H1+J)*3.14159/180
1250 S1=S1+2*P1
1260 S2=S2+2*F3
1270 NEXT W
1280 R=U(1,I)
1290 B=U(2,I)
1300 GOSUB 600
1301 U(3,I)=(H1+J)*3.14159/180
1310 S1=(S1+P1)*(U(1,I)-U(1,1))/(2*I)
1320 S2=(S2+F3)*(U(1,I)-U(1,1))/(2*I)
1330 N=I
1340 PRINT
1350 PRINT 'TOTAL POWER FACTOR=';S1,'TOTAL FORCE FACTOR=';S2
1351 G$='O'
1352 PRINT
1353 PRINT 'ENTER ''EVAL'' IF YOU WISH TO EVALUATE THIS DESIGN'
1354 PRINT 'FOR A RANGE OF PITCHES'
```

```
1355 INPUT G$
1356 IF G$='EVAL' THEN 1580
1360 GO TO 60
1365 PRINT 'PITCH?'
1366 INPUT P
1370 PRINT 'RADIUS FROM, TO, INCREMENT'
1380 INPUT R1,R2,R3
1390 PRINT 'ROOT CHORD,TIP CHORD'
1400 INPUT D1,D2
1401 M=(D2-D1)/(R2-R1)
1402 X=D1-(R1*(D2-D1))/(R2-R1)
1410 Z=1
1415 I=1
1420 FOR R=R1 TO R2 STEP R3
1430 B=M*R+X
1434 U(1,I)=R
1435 U(2,I)=B
1440 GOSUB 600
1441 U(3,I)=(H1+J)*3.14159/180
1442 I=I+1
1450 S1=S1+P1
1460 S2=S2+F3
1470 IF R =R1 THEN 1510
1480 IF R=R2 THEN 1510
1490 S1=S1+P1
1500 S2=S2+F3
1510 NEXT R
1520  O=(R2-R1)/R3+1
1530 S1=S1*(R2-R1)/(2*O)
1540 S2=S2*(R2-R1)/(2*O)
1545 N=(R2-R1)/R3+1
1550PRINT
1560 PRINT 'TOTAL POWER FACTOR=';S1,'TOTAL FORCE FACTOR=';S2
1562 PRINT
1563 PRINT 'ENTER ''EVAL'' IF YOU WISH TO EVALUATE THIS DESIGN'
1564 PRINT 'FOR A RANGE OF PITCHES'
1565 INPUT G$
1566 IF G$='EVAL' THEN 1580
1570 GO TO 60
1580 PRINT
1590 PRINT 'PITCH FROM, TO, INCREMENT?'
1600 INPUT P1,P2,P3
1601 IF P1=0 THEN 1930
1602 IF P1<0 THEN 60
1610 PRINT USING 1620,'PITCH','POWER FACTOR','FORCE FACTOR'
1620 FORM SKIP2,C,X7,C,X3,C
1625 PRINT
1630 FOR P=P1 TO P2 STEP P3
1634 S1=0
1635 S2=0
1640 FOR I=1 TO N
1650 A=U(1,I)*ATN(1)*4
1660 D=2*A/P
1670 C=4.0*((2*A)**2)/(U(2,I)*SQR((2*A)**2+P**2))
1680 J=(U(3,I)-ATN(D))*180/(4*ATN(1))
1682 IF J>-5 THEN 1684
1683 J=-5
```

```
1684 IF J<=25 THEN 1690
1685 J=25
1690 F1=C0*J**9+C9*J**8+C8*J**7+C7*J**6+C6*J**5+C5*J**4
1700 F1=F1+C4*J**3+C3*J**2+C2*J+C1
1710 T=F1/(F1/J+C)
1720 H1=ATN(D)+T
1724 T1=T*180/(4*ATN(1))
1730 J=(U(3,I)-H1)*180/(4*ATN(1))
1732 IF J>-5 THEN 1734
1733 J=-5
1734 IF J<=25 THEN 1740
1735 J=25
1740 F1=C0*J**9+C9*J**8+C8*J**7+C7*J**6+C6*J**5+C5*J**4
1750 F1=F1+C4*J**3+C3*J**2+C2*J+C1
1760 F2=B0*J**9+B9*J**8+B8*J**7+B7*J**6+B6*J**5+B5*J**4
1770 F2=F2+B4*J**3+B3*J**2+B2*J+B1
1775 T1=U(3,I)-H1-J*3.14159/180
1776 T1=COS(T1)
1780 P1=(F1*COS(H1)-F2*SIN(H1))*T1*((2*A)**2+P**2)*A*U(2,I)/(2*P**3)
1790 F3=(F1*SIN(H1)+F2*COS(H1))*T1*((2*A)**2+P**2)*U(2,I)/(2*P**2)
1795 H7=H1*180/3.14159
1800 S1=S1+P1
1810 S2=S2+F3
1820 IF I=1 THEN 1860
1830 IF I=N THEN 1860
1840 S1=S1+P1
1850 S2=S2+F3
1860 NEXT I
1870 S1=S1*(U(1,N)-U(1,1))/(2*N)
1880 S2=S2*(U(1,N)-U(1,1))/(2*N)
1890 PRINT USING 1900,P,S1,S2
1900 FORM PIC(ZZ.##),X2,PIC(ZZZZZZZZ.##),X4,PIC(ZZZZZZZZ.##)
1905 PRINT
1910 NEXT P
1920 GO TO 60
1930 END
```

***************WEP***************

```
10 DIM U(3,40)
15 P9=4*ATN(1)
20 READ C0,C9,C8,C7,C6,C5,C4,C3,C2,C1
30 READ B0,B9,B8,B7,B6,B5,B4,B3,B2,B1
40 PRINT 'THIS PROGRAM EVALUATES A GIVEN WINDMILL FOR'
50 PRINT 'ALL PERFORMANCE AND STRUCTURAL VARIABLES'
60 PRINT 'OVER A RANGE OF WIND SPEEDS, PITCHES, OR'
70 PRINT 'ROTATION SPEEDS'
80 PRINT '  ALL INPUT MUST BE IN UNITS OF FEET AND/OR'
81 PRINT 'SECONDS, UNLESS OTHERWISE REQUESTED'
90 PRINT 'ENTER WINDMILL SPECS: BLADE RADIUS, CHORD, ANGLE'
100 PRINT 'TERMINATE INPUT WITH ZERO RADIUS'
101 DATA 3.524514E-12,-6.30564E-10,4.417507E-8,-1.499429E-6
102 DATA 2.492219E-5,-1.614988E-4,-4.649162E-4,8.909225E-3
103 DATA 7.790631E-2,5.672455E-2
104 DATA -8.995701E-12,1.026354E-9,-4.779145E-8,1.167798E-6
```

```
105 DATA -1.607709E-5,1.247505E-4,-5.00344E-4,8.673142E-4
106 DATA -4.246052E-4,1.005569E-2
110 I=1
120 PRINT
130 PRINT 'RADIUS, CHORD, ANGLE?'
140 INPUT U(1,I),U(2,I),U(3,I)
150 IF U(1,I)=0 THEN 175
160 I=I+1
170 GO TO 130
175 N=I-1
180 PRINT
190 PRINT 'WHAT DO YOU WISH TO VARY: WIND, PITCH, OR RPM?'
195 PRINT 'ENTER ZERO TO TERMINATE PROGRAM'
200 INPUT A$
210 IF A$='WIND' THEN 350
220 IF A$='RPM' THEN 470
225 IF A$='0' THEN 1500
230 PRINT
240 PRINT 'WIND SPEED (MPH)?'
250 INPUT V1
260 PRINT 'PITCH FROM, TO, INCREMENT?'
270 INPUT P1,P2,P3
280 PRINT
290 GOSUB 620
300 FOR P=P1 TO P2 STEP P3
310 GOSUB 690
320 GOSUB 1340
330 NEXT P
340 GO TO 180
350 PRINT
360 PRINT 'PITCH?'
370 INPUT P
380 PRINT 'WIND SPEED (MPH) FROM, TO, INCREMENT?'
390 INPUT V2,V3,V4
400 PRINT
410 GOSUB 620
420 FOR V1=V2 TO V3 STEP V4
430 GOSUB 690
440 GOSUB 1340
450 NEXT V1
460 GO TO 180
470 PRINT
480 PRINT 'WIND SPEED (MPH)?'
490 INPUT V1
500 PRINT 'RPM FROM, TO, INCREMENT?'
510 INPUT R7,R8,R9
520 P1=V1*88/R7
530 P2=V1*88/R8
540 P3=V1*88/R9
550 PRINT
560 GOSUB 620
570 FOR P=P1 TO P2 STEP P3
580 GOSUB 690
590 GOSUB 1340
600 NEXT P
610 GO TO 180
```

```
620 PRINT USING 630, 'RADIUS','CHORD','ANGLE'
630 FORM SKIP2,C,X3,C,X3,C
640 FOR I=1 TO N
645 PRINT
650 PRINT USING 660,U(1,I),U(2,I),U(3,I)
660 FORM PIC(ZZ.###),X2,PIC(ZZ.###),X3,PIC(ZZ.###)
670 NEXT I
680 RETURN
690 S1=0
700 S2=0
710 S3=0
720 FOR I=1 TO N
730 A=U(1,I)*P9
740 D=2*A/P
750 C=4.0*((2*A)**2)/(U(2,I)*SQR((2*A)**2+P**2))
760 J=U(3,I)-ATN(D)*180/P9
770 IF J>-5 THEN 790
780 J=-5
790 IF J<=25 THEN 810
800 J=25
810 M=(V1*44/30)*SQR(P**2+(2*A)**2)/(1117*P)
820 F2=.0001
830 F1=C0*J**9+C9*J**8+C8*J**7+C7*J**6+C6*J**5
840 F1=F1+C5*J**4+C4*J**3+C3*J**2+C2*J+C1
845 IF M<0.3 THEN 850
846 GOSUB 1150
850 T=F1/(F1/J+C)
860 H1=ATN(D)+T
870 J=(U(3,I)*P9/180-H1)*180/P9
890 IF J>-5 THEN 910
900 J=-5
910 IF J<=25 THEN 940
920 J=25
940 F1=C0*J**9+C9*J**8+C8*J**7+C7*J**6+C6*J**5
950 F1=F1+C5*J**4+C4*J**3+C3*J**2+C2*J+C1
960 F2=B0*J**9+B9*J**8+B8*J**7+B7*J**6+B6*J**5
970 F2=F2+B5*J**4+B4*J**3+B3*J**2+B2*J+B1
975 IF M<0.3 THEN 980
976 GOSUB 1150
980 t1=cos((u(3,i)-j)*p9/180-h1)
990 P1=(F1*COS(H1)-F2*SIN(H1))*T1*((2*A)**2+P**2)*A*U(2,I)/(2*P**3)
1000 F3=(F1*SIN(H1)+F2*COS(H1))*T1*((2*A)**2+P**2)*U(2,I)/(2*P**2)
1010 F4=F3*U(1,I)
1020 S1=S1+P1
1030 S2=S2+F3
1040 S3=S3+F4
1050 IF I=1 THEN 1100
1060 IF I=N THEN 1100
1070 S1=S1+P1
1080 S2=S2+F3
1090 S3=S3+F4
1100 NEXT I
```

```
1110 S1=S1*(U(1,N)-U(1,1))/(2*N)
1120 S2=S2*(U(1,N)-U(1,1))/(2*N)
1130 S3=S3*(U(1,N)-U(1,1))/(2*N)
1140 RETURN
1150 REM MACH CORRECTION
1160 M0=-0.25*F1+0.525
1170 IF M<=M0 THEN 1280
1180 IF M<=0.95 THEN 1210
1181 PRINT
1182 PRINT 'VALUES FOLLOWING THIS STATEMENT ARE'
1183 PRINT 'BEYOND MACH CORRECTION RANGE.  ERROR'
1184 PRINT 'MAY BE UNACCEPTABLE'
1190 M=0.95
1210 W1=(-0.339105*F1-0.34387)*(0.95-M0)+F1
1220 L=SQR((0.95-M0)**2+(W1-F1)**2)
1230 A1=(-0.339105*F1-0.34387)*(M-M0)+F1
1240 IF ((M-M0))/(0.95-M0)>(2/3) THEN 1270
1250 F1=A1+0.15*SIN((3*P9/(2*(M1-M0)))*(M-M0))
1260 GO TO 1280
1270 F1=A1-(0.5*F1+0.05)*SIN((3*P9/(M1-M0))*(M+(2*M1-M0)/3))
1280 M0=-0.0125*J+0.6
1290 IF M>M0 THEN 1330
1300 IF M<=0.95 THEN 1320
1310 M=0.95
1320 F2=(-9.62E-5*J+3.149E-3)*(10*(M-M0))**3+F2
1330 RETURN
1340 REM PERFORMANCE CALCULATE AND OUTPUT
1350 P4=0.002378*((V1*22/15)**3)*S1
1355 PRINT
1356 PRINT
1357 PRINT
1360 D2=0.002378*S2*((V1*22/15)**2)
1370 M3=0.002378*S3*((V1*22/15)**2)
1380 S5=(SQR((2*P9*U(1,N))**2+P**2))/P
1390 Q=P4*P/(P9*V1*44/15)
1400 D1=D2/Q
1410 E=S1*27/(16*P9*U(1,N)**2)
1420 E1=S1/(P9*U(1,N)**2)
1421 PRINT 'WIND SPEED ';V1;' MPH        RPM ';((V1*44/30)/P)*60
1422 PRINT 'PITCH ';P;' FEET      TIP SPEED RATIO ';S5
1430 PRINT 'POWER OUTPUT ';P4;' FT-LB/SEC'
1440 PRINT 'THRUST ';D2;' LB'
1450 PRINT 'TORQUE ';Q;' FT-LB'
1460 PRINT 'TOTAL HUB MOMENT ';M3;' FT-LB'
1470 PRINT 'BETZ EFFICIENCY ';E
1480 PRINT 'ABSOLUTE EFFICIENCY ';E1
1490 RETURN
1500 END
```

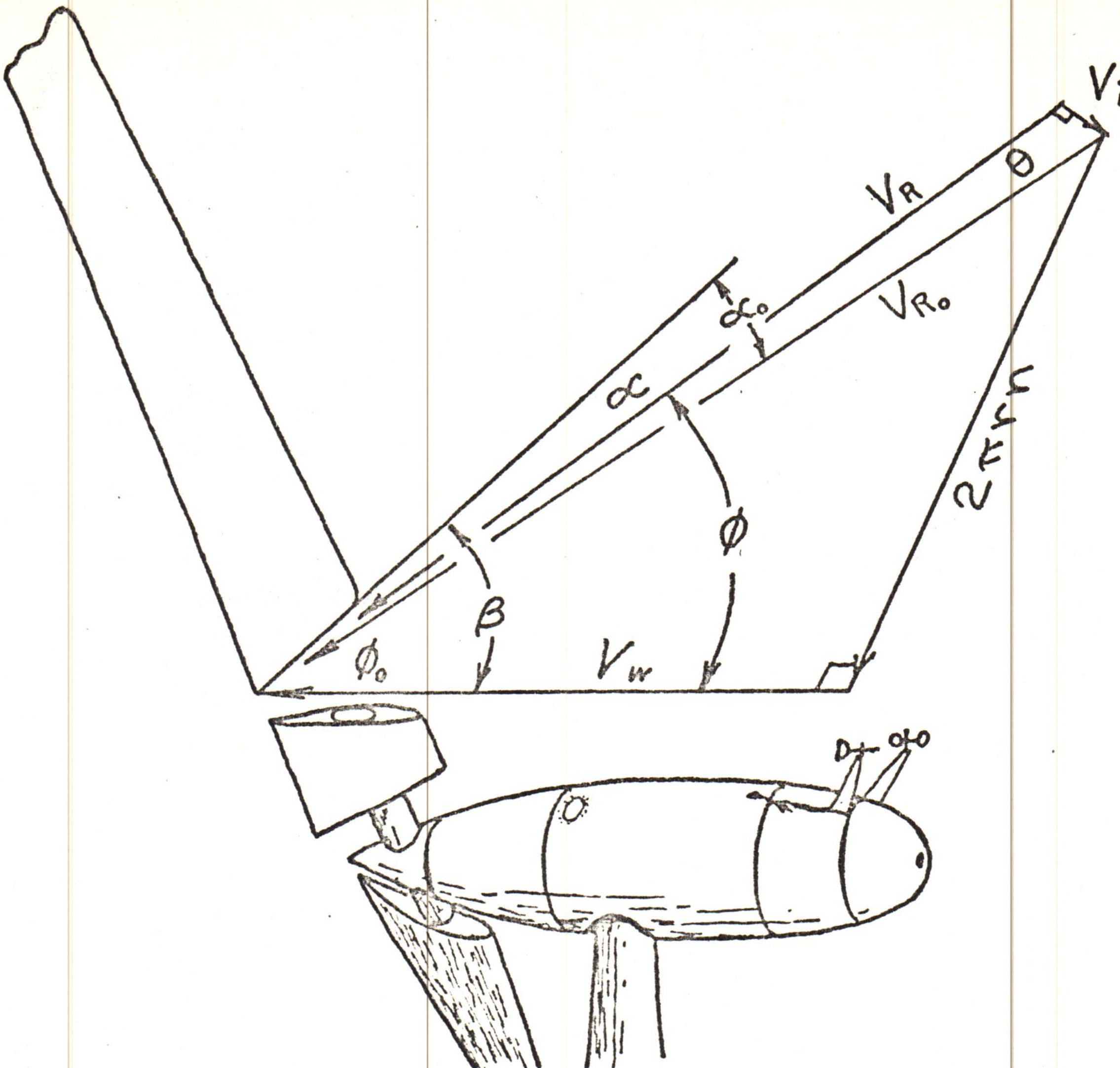

Figure 1. Velocity Vectors

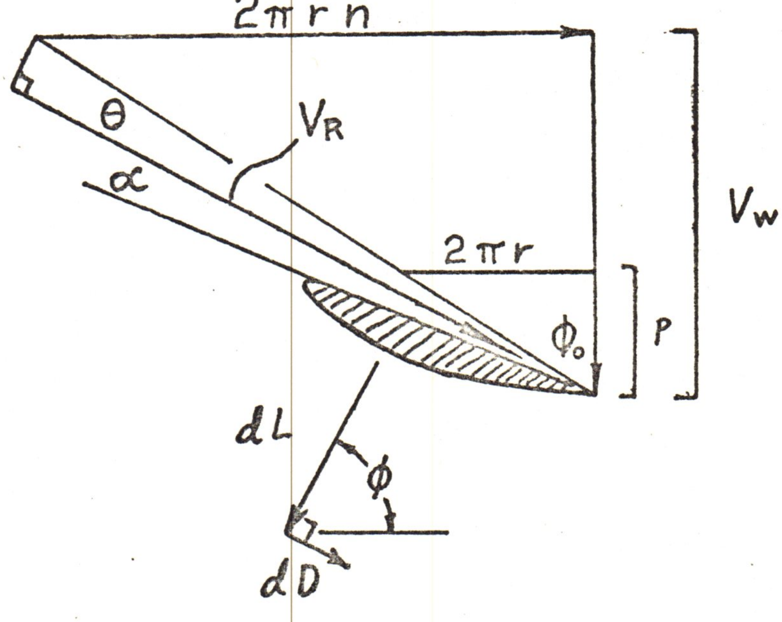

Figure 2. Vortex Relations with Pitch

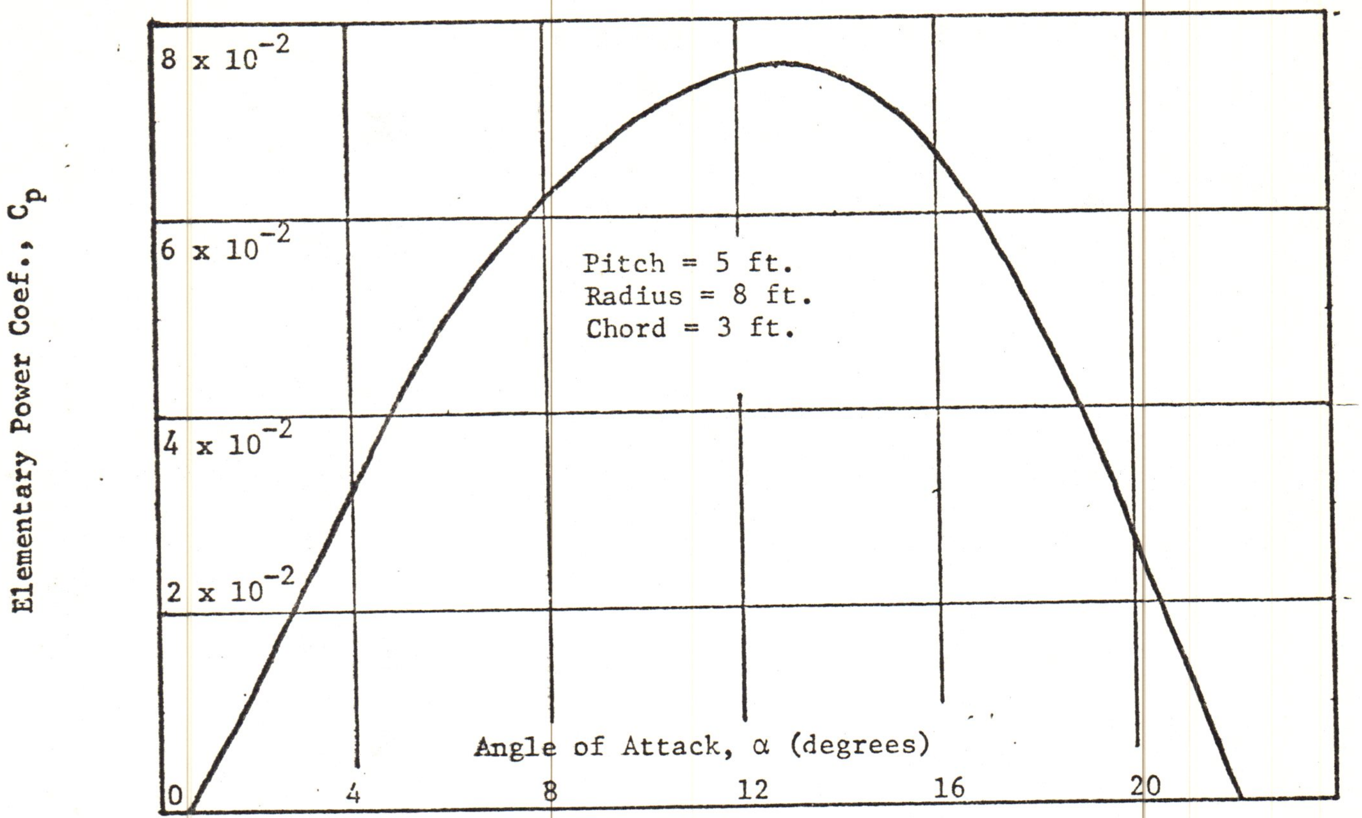

Figure 3. Power and Angle of Attack

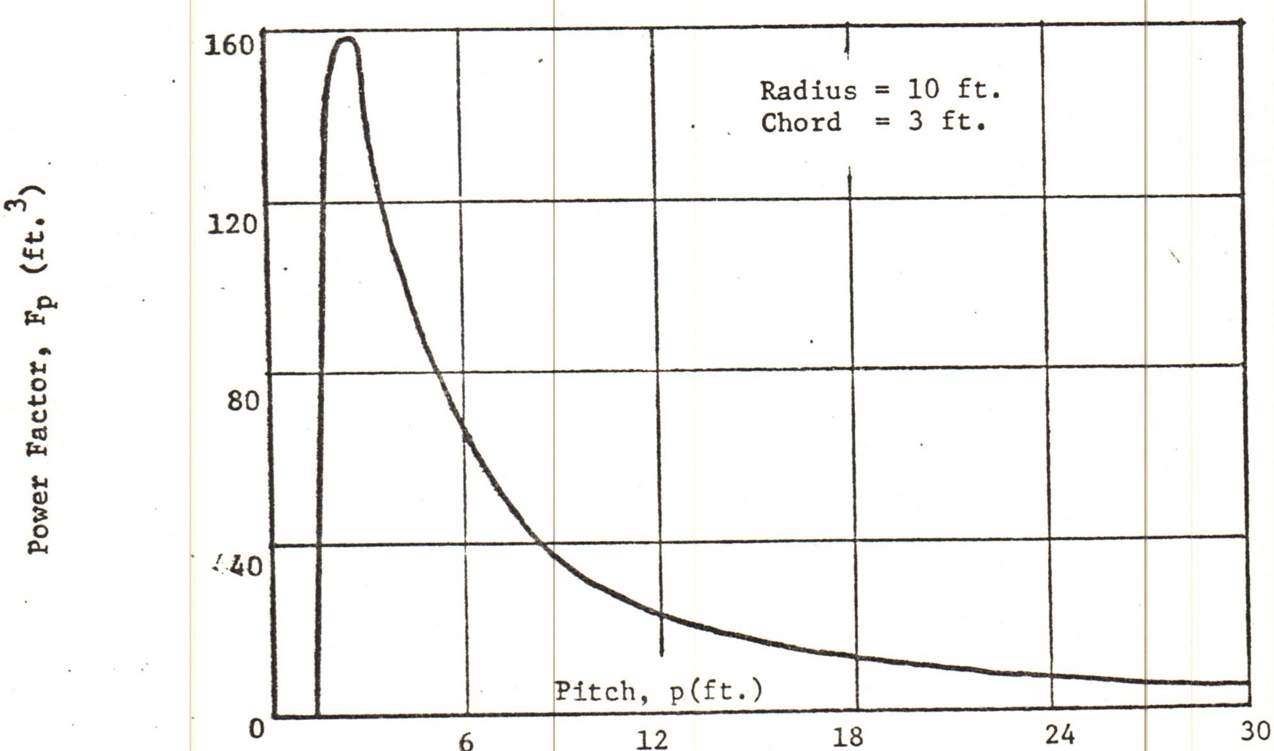

Figure 4. Power and Pitch

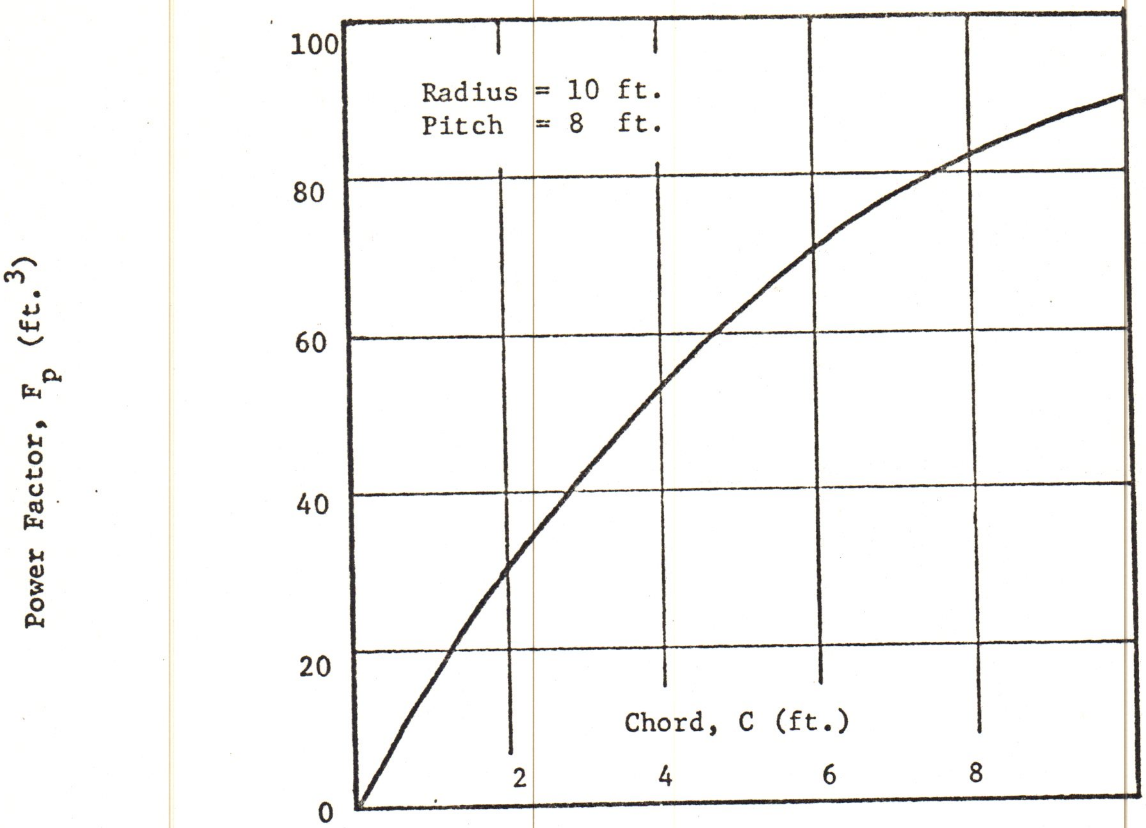

Figure 5. Power and Chord

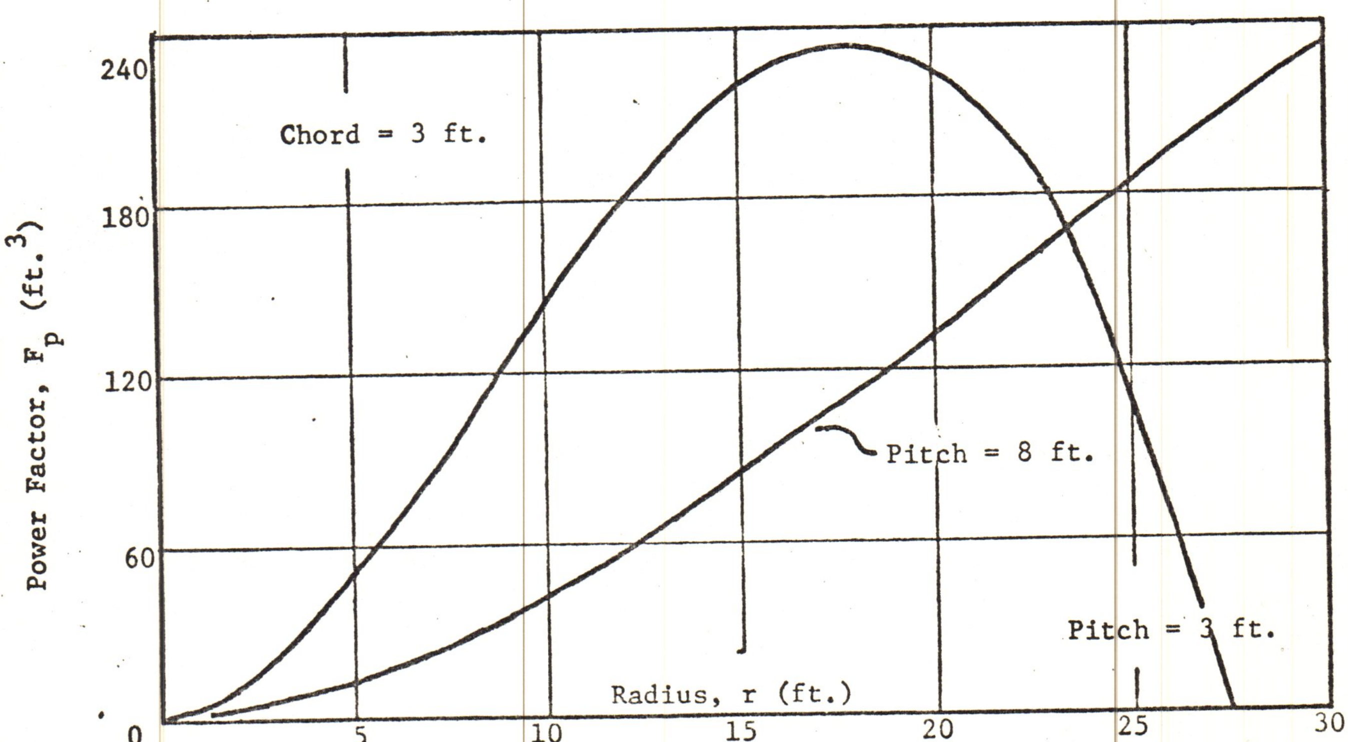

Figure 6. Power and Radius

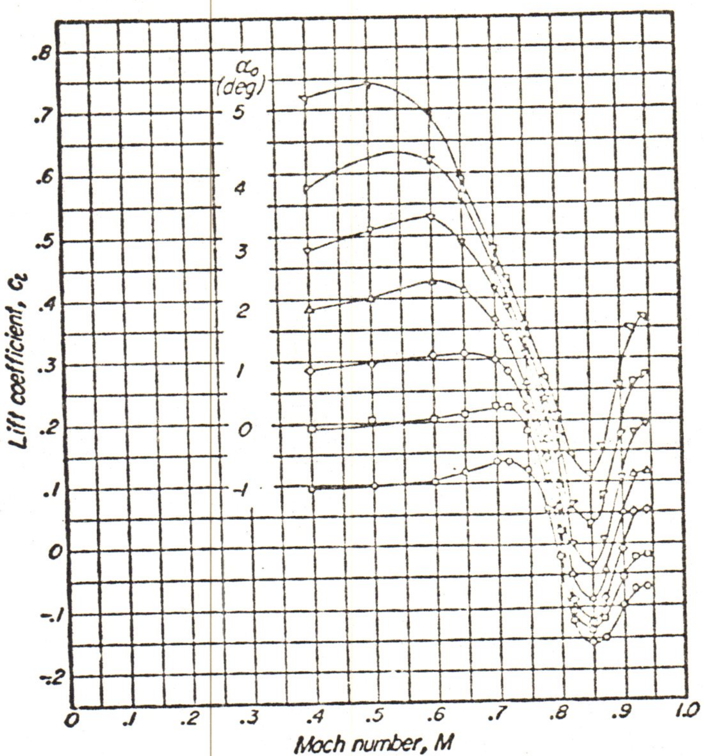

Figure 7. C_L and Mach Number

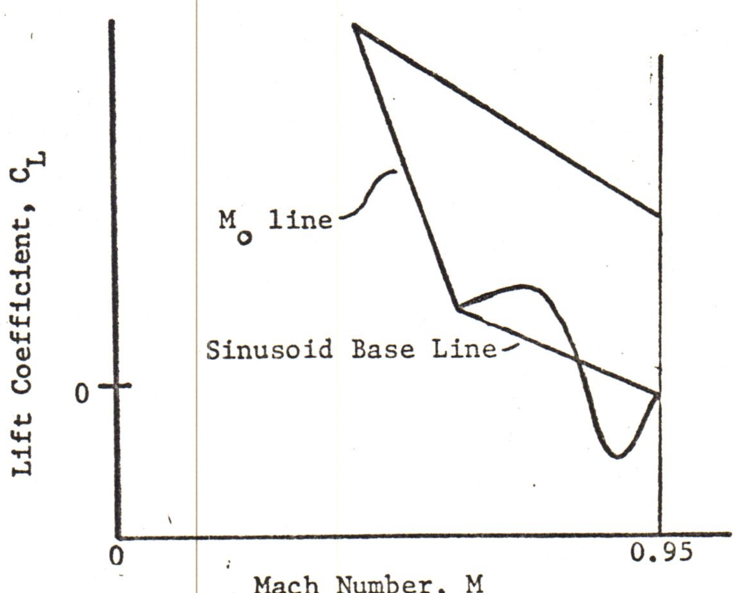

Figure 8. Correction Function

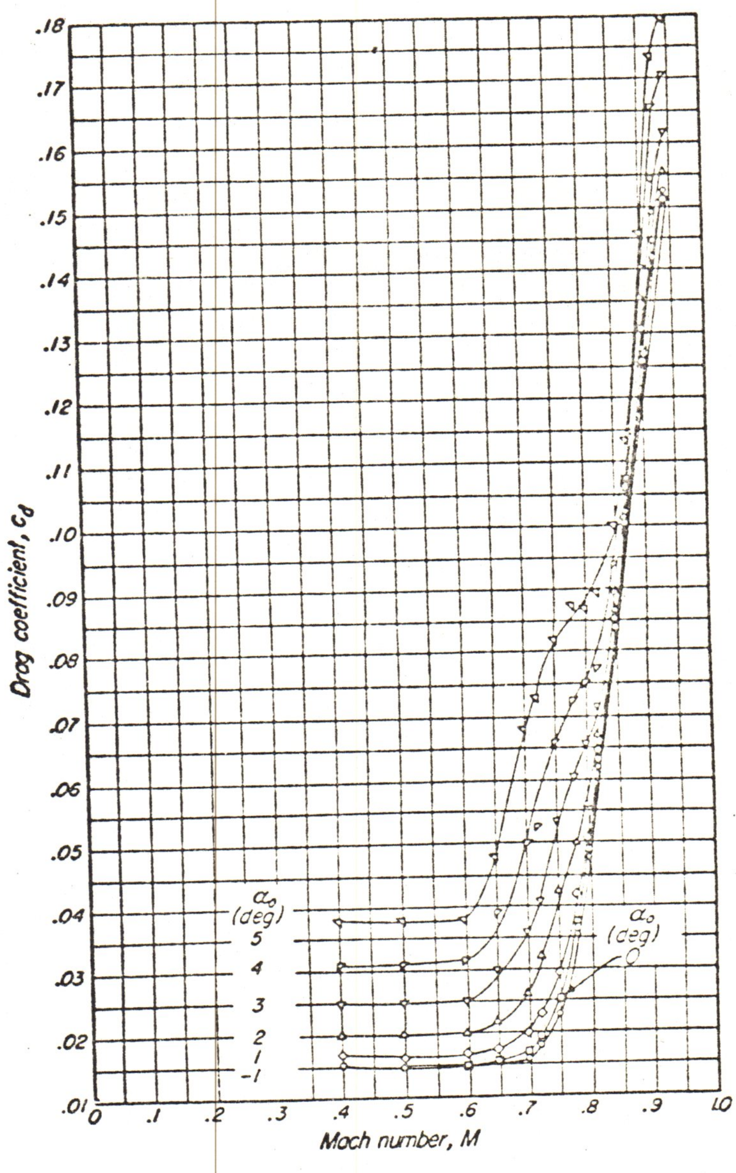

Figure 9. C_D and Mach Number